



## Fuzzy logic modelling tracer response in milli torus reactor under aerated and non-aerated conditions

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### ARTICLE INFO

#### Article history:

Received 11 January 2009

Received in revised form 17 May 2009

Accepted 18 May 2009

#### Keywords:

Fuzzy logic modelling

MISO system

Milli torus reactor

Tracer response

Mixing

### ABSTRACT

The present paper aims at developing and modelling the dynamic process of tracer response in aerated and non-aerated conditions, using a fuzzy logic approach, for such case the milli loop geometry is operated under turbulent flow regime. An identification of fuzzy logic controller whose rules approximated the measured data is first developed. The second-order differential equation is used to identify significant parameters of fuzzy logic controller. A fuzzy model is built with multiple inputs, single output (MISO) non-linear dynamic system under Matlab<sup>®</sup>/Simulink environment. The proposed technique takes into account the physical description of the tracer response curve. The selection of the variables and the shape of the membership functions are discussed. The results of the model are in good agreement with the experimental data.

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### 1. Introduction

Two approaches are often used to describe mixing inside the reactors. The first one is based on the computational fluid dynamic (CFD) which is the most common technique used to solve the equations of fluid motion on digital computers. However, this finite element method requires a long computation time [1]. The second one is the classical approach based on the experimental tracer studies. Ranade [2] reported that this practice provides detailed local information about turbulence and mixing; which may ultimately determine the reactor performances.

Generally, in gas–liquid reactors, the liquid phase is a continuous phase while the gas bubbles are dispersed within several types of reactors (airlifts, bubble columns, stirred tanks and torus reactors). The gas–liquid milli torus reactor is characterised by fluid circulation in defined loop geometry. In this reactor, the mixing is enhanced by both impeller rotation speed and gas. Often in a one phase flow, the torus reactor is divided into two regions: near the axial flow impeller where a complex swirl flow is induced and outside the impeller swept region where the decay swirl intensity is observed. In the case of a two-phase flow, the energy required for the movement of the fluid is focally introduced at two points in the reactor, via the impeller and the sparger. The nature of the latter is of less importance if the design is such that gas is effectively cap-

tured and dispersed by the impeller [3]. It can be operated in two flow regimes: not-dispersed flow regime and dispersed flow regime [4] depending on different parameters, including operating conditions, type and location of sparger, turbulence fields and properties of liquid phase. It can offer some flexibility in operation and it can be operated in batch as well as in continuous mode. The work of Gavrilescu and Tudose [5] indicated that knowledge of the liquid RTD is a parameter of particular importance for an accurate kinetic modelling of the system; it remains a useful tool in reactor design to achieve or preserve a desired flow pattern. This is important during the process of scale-up from laboratory-scale to industrial-scale reactors.

There are different kinds of reactor models depending on whether flow is close to plug, mixed, or somewhere in between [6]. A simple and appropriate reactor model which adequately represents the physical phenomena occurring inside the reactor is often chosen. Also, the tank in series model and the dispersion model give essentially the same results, provide that the number of tanks or the Peclet number is selected appropriately. The choice between both models is a matter of personal preference and computational convenience [7]. In single liquid phase, Benkhelifa et al. [8] modelled the batch torus reactor using axial dispersed plug flow with total recirculation in order to estimate the mean circulation time and the axial dispersion coefficient. They found that the value of the axial dispersion coefficient is rather small and constant in the whole range of mixing Reynolds number studied in their work. Zhang et al. [9] developed a mathematical model for airlift loop reactor by applying the axial dispersion model to the riser and down-comer,

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**Nomenclature**

$B_o$	Bodenstein number for the liquid phase, dimensionless
$C$	tracer concentration ( $\text{mol l}^{-1}$ )
$C_o$	final tracer concentration ( $\text{mol l}^{-1}$ )
$d_t$	inner diameter of cross section of the milli reactor (m)
$E_z$	axial dispersion coefficient ( $\text{m}^2 \text{s}^{-1}$ )
$j$	integer, dimensionless
$K$	constant
$k$	amplitude corresponding to the mixing time, dimensionless
$ksi$	damping coefficient, dimensionless
$L_t$	mean length of the milli reactor (m)
$N$	impeller rotation speed ( $\text{s}^{-1}$ )
$R_t$	bending of reactor radius (m)
$t$	time (s)
$t_c$	cycling time (s)
$t_m$	mixing time (s)
$U_g$	superficial gas velocity ( $\text{m s}^{-1}$ )
$V_L$	mean liquid circulation velocity ( $L_t/t_c$ ) ( $\text{m s}^{-1}$ )
$V_R$	volume of the reactor ( $\text{m}^3$ )
$wn$	undamped pulsations or peaks number in term of mixing process, dimensionless
$y$	the final output value of the model, dimensionless
$Z$	the geometrical distance between injection and detection (m)
<i>Greek Symbols</i>	
$\alpha$	pitch angle of impeller, degree
$\beta$	constant
$\theta$	dimensionless time ( $t/t_c$ )
$\mu_{A_i}$	the membership function of the antecedent linguistic variables $A_i$ , dimensionless

respectively. Their model is based on the physics of an airlift loop reactor. Also, the tracer impulse can be considered to flow in an infinite tube that consists of alternating riser, gas–liquid separator and down-comer. Claudel et al. [10] reported that four methods are well adapted to simulate the residence time distribution: the expert system, the neural network, the fuzzy logic and the possibility logic theory. They used the possibility theory developed by Zadeh [11], who combined fuzzy sets and probability theory. The present work deals with modelling the dynamic tracer response in torus reactor under aerated and non-aerated conditions using fuzzy logic. This tool has been successfully applied to control dynamic illuminance for solar radiation [12] and with its combination with mechanistic model, lead to hybrid fuzzy-mechanistic used for modelling algal concentration [13].

The fuzzy logic is widely applied in many fields of our daily life, in many commercial products, such as cameras, washing machines, microwaves, cars, battery chargers, motorway tunnel air conditioning, trip planning and in many engineering applications such as: system identification, computer vision, automatic control, wastewater treatment [14] biomedical instrumentation and many more. It is proposed by Zadeh [15] with the main idea of providing effective mean to approximate the real behaviour of systems which are too complex [16]. For that, the description of the system uses linguistic variables (natural language) which play a key role in most of the applications of fuzzy logic [17]. Unlike the classical logic (Boolean logic), which only admits discrete values at their extremes, i.e. zero (totally false) and one (totally true), fuzzy logic admits a continuous variation values from zero to one [15,18]. Therefore, it is an

extension of the classic systems. Fuzzy logic deals with the concept of a fuzzy set and measures the degree to which the event may occurs while probability predicts unknown outcomes based on known parameters. Usually, it means almost fuzzy logic subsets.

Nowadays, more attention has been paid to the multiple model approach for non-linear systems. Fuzzy logic seems to be a useful tool for modelling highly complex systems whose behaviours are not well understood. A good model is one which adequately explains the experimental data, by a selection from a set of test rules including corresponding input variables. The rules must be combined in some way which allows obtaining accurate results. So, the order in which the rules are executed is not important. According to Kuo and Lin [19] there are two difficulties in designing any fuzzy logic systems: the shape of the membership functions and the choice of the fuzzy rules which can define the variables to be used and the ways in which the rules have to be combined.

The objective of fuzzy logic is to explain the relationships between inputs and outputs data and then estimates the parameters of the model. For doing this, it is important to have a list of if-then statements, called rules. The structure of fuzzy system, which includes an implication between actions and conclusions are given in the form: If premise then conclusion.

There are two types of fuzzy inference systems: Mamdani-type and Takagi–Sugeno-type (TS) fuzzy systems [20,21]. The difference between both is the consequence (fuzzy implication statement) of the fuzzy rules. Castro and Delgado [22] and Ying [23] reported that Mamdani fuzzy systems use fuzzy sets as rules consequence whereas TS fuzzy systems employ linear functions of input variables as rule consequence in an iterative manner.

Fuzzy logic in chemical engineering process opens a new way to represent knowledge and to deal with problems encountered in chemical engineering process and related fields.

Several steps are generally used for solving fuzzy problems:

- Define the fuzzy problem
- Specify input and output variables and their ranges
- Draw an appropriate membership profiles for each variable range
- Determine rules
- Select an appropriate defuzzification method
- Begin to test the system.

The present paper deals with modelling the dynamic process of tracer dispersion obtained in aerated and non-aerated conditions in milli loop geometry. The milli torus reactor is operated under turbulent flow regime. A simulator is developed under Matlab®/Simulink environment. The model takes into account a list of rules, the level of activation of each rule, the second-order differential equation and the parameters able to affect the tracer response curves. As far as we know, no study has been devoted to the identification of MISO (multi inputs–single output) fuzzy models from input–output tracer response curves in aerated and non-aerated conditions. The model is built with two inputs (impeller rotation speed and superficial gas velocity) and one output (RTD curves).

## 2. Basic governing equations

Let us assume that we have a collection of  $i$ th fuzzy rules,  $R_i$ :

$$R_i : \text{if } x \text{ is } A_i \text{ then } y_i = f_i(x) \quad i = 1, \dots, r; \quad r \in \mathbb{N}^* \quad (1)$$

Where:

- $x = (x_1, \dots, x_p)$  the input variables,  $p \in \mathbb{N}^*$
- $y_i \in \mathbb{R}$  is the output variables.

The membership function of the antecedent linguistic variables  $A_i$  is commonly denoted by  $\mu_{A_i}(x)$  and usually has the form:

$$\mu_{A_i}(x) : \mathbb{R}^p \rightarrow [0, 1], p \in \mathbb{N}^*$$

Depending on a chosen set of parameters, each function  $f_i$  realizes a second-order ordinary differential equation. The set of function  $f_i$  are chosen in such a way as to well describe the dynamic model governing the dispersion of inert tracer through milli torus reactor:

$$(E) a_2^{(i)} \frac{d^2 y_i}{dt^2} + a_1^{(i)} \frac{dy_i}{dt} + a_0^{(i)} y_i = b_0^{(i)} \exp(t) \quad \forall i, i = 1, \dots, r \quad (2)$$

where the constants coefficients  $a_j^{(i)}, b_0^{(i)}$  are termed scalar numbers for each rule  $i$ , for  $i = 1, \dots, r$ .

The equation above is known as linear and is among the easiest to solve. Many engineering as well as non-engineering systems can be modelled by this equation.

When  $b_0^{(i)} = 0$  for  $i = 1, \dots, p$ , Eq. (2) is known as the homogeneous or complementary equation.

$$Ri : \text{if } x_i \text{ is } A_i \text{ then } a_2^{(i)} \frac{d^2 y_i}{dt^2} + a_1^{(i)} \frac{dy_i}{dt} + a_0^{(i)} y_i = 0 \quad \forall i, i = 1, \dots, r \quad (3)$$

The homogeneous model allows the conception of the controller as well as the analysis of the stability of the system because the model becomes linear. However in the case where  $a_i = 0, \forall i, i = 1, \dots, r$ , the model (E) can be written as:

$$(S) R_i : \text{if } x_i \text{ is } A_i \text{ then } b_0^{(i)} \exp(t) = 0 \quad (4)$$

Two cases appear:

- (a) if  $b_0^{(i)} = 0$  then (S) is a tautology
- (b) if  $b_0^{(i)} \neq 0$  then (S) is verified only when  $t \rightarrow -\infty$  which is absurd.

In the following, we suppose that  $a_i \neq 0 \forall i, i = 1, \dots, r$ .

In this paper, the Laplace transform has been introduced in order to simplify the solution of linear differential equation. The analysis of the system dynamic behaviour can be carried out by solving Eq. (2) for impulse response in the Laplace domain which leads, with respect to the transform parameter  $p$ , to:

$$y(p) = \frac{k}{1 + 2ksi/wnP + 1/wn^2 P^2} E(p) \quad (5)$$

where:  $k$ : amplitude corresponding to the mixing time;  $wn$ : undamped pulsations or peaks number in term of mixing process;  $ksi$ : damping coefficient.

To infer the output, the accomplishment degree is first calculated for each rule which is equal to the degree of membership for each input value.

$$i.e. \beta_i = \mu_{A_i}(x) \quad (6)$$

So, the final output value of the model is given by the weighted-average method. This can be written as:

$$y = \frac{\sum_{i=1}^{i=r} \beta_i(x) y_i}{\sum_{i=1}^r \beta_i(x)} \quad (7)$$

A more comprehensive way used to study the mixing inside the reactors is the tracer technique. When an instantaneous pulse of tracer is injected from the impulse injection port, the outlet stream is monitored as a function of time and corresponds to the superimposition of a periodic function on an exponential decrease as previously shown by Belleville et al. [24] and Benkhefifa et al. [8]. According to Levenspiel [6] the tracer starts spreading and this can be caused by three mechanisms: velocity profile, turbulent mixing and molecular diffusion. The contribution of molecular diffusion

**Table 1**  
Variables for the fuzzy logic system.

Variable	Name	Unit	Fuzzy	Nomenclature	Range
1	Superficial aeration velocity	cm/s	Very low	(VLO)	0
			Low	(LO)	0–0.02
			Medium	(MD)	0.01–0.03
			High	(HG)	0.02–0.05
			Very high	(VHG)	0.04–0.06
2	Impeller rotation speeds	rpm	Higher	(HGH)	0.05–0.09
			Very low	(VLO)	0–300
			Low	(LO)	200–400
			Medium	(MD)	300–500
			High	(HG)	400–600
	Very high	(VHG)	500–700		
	Higher	(HGH)	600–900		

is usually negligible under turbulent flow conditions which are well known enhance mixing [25]. Generally, two parameters can affect the tracer response profile quality. The first one corresponds to the tracer injecting position which should not be too close to the platinum microprobe because the flow is not well developed radially when it moves the section where the probe is located. The second one concerns the data acquisition interval. In this paper, the data acquisition interval of 0.01 s is found appropriate for our experiments. Several mixing models are proposed. They reduce the given information by the tracer response to a small number of parameters, which can later be used in design and scale-up [26]. It is necessary to select the information describing the process in order to model it. The following parameters have been chosen: peaks number, time response for the first peak, circulation time, equilibrium concentration etc., which describe our real flow systems (Fig. 1). Thus, the proposed fuzzy logic model is reduced to the above parameters for modelling tracer response inside the milli torus reactor, which can later be used in design and scale-up.

### 2.1. Structure of the fuzzy system

For a single system the input variable chosen is ( $N$ ); whereas for two-phase systems, two inputs are selected ( $N$  and  $Ug$ ). These inputs are processed by the Mamdani fuzzy inference method, allowing the identification of the output. The range assigned for each input is an important detail for establishing the total number of rules and the accuracy of results. The determination of rules in the case of tracer response is an important way to understand the problem of the macromixing for single and two-phase systems and to assess the behaviour of the system using multiple input variables.

Table 1 gives the input variables and the fuzzy nomenclature for describing the system.

Generally, the fuzzy logic system is divided into three operations: fuzzification, inference and defuzzification. The fuzzification step allows translating the real input values into linguistic variables by using fuzzy sets theory. The fuzzy inference infers with the fuzzy rule base in computational terms. The results of the fuzzy inference are translated into output values via defuzzification block. The latter is an important step because the output result may or may not be successful. These operations are shown in Fig. 2 which provides two inputs, one output.

### 3. Experimental apparatus

The experiments are carried out on a laboratory-scale milli torus reactor vertically set, made of Pyrex of 23.4 mm in diameter and a volume of 0.14 L. The scheme of the experimental set-up is shown

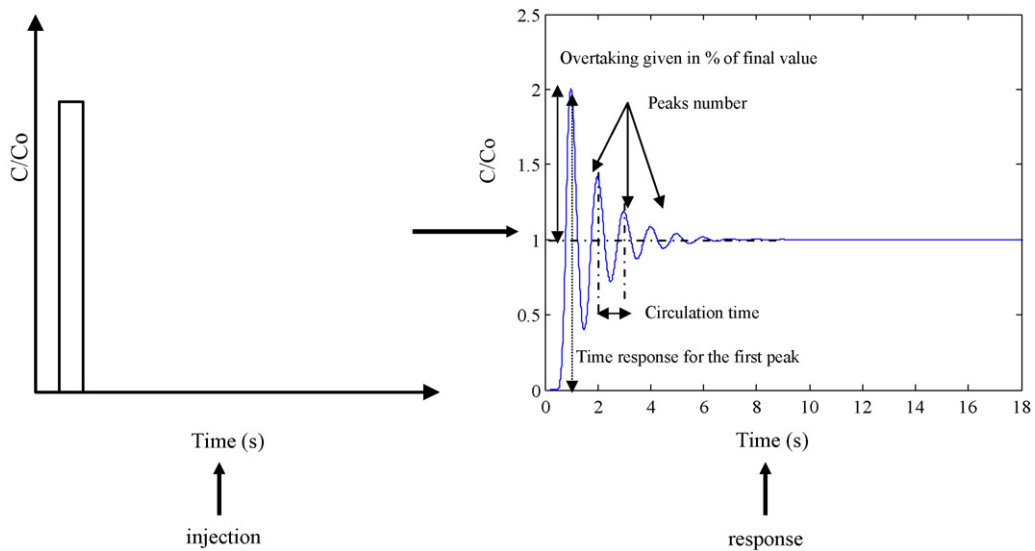


Fig. 1. Characteristics of injection and tracer response curve.

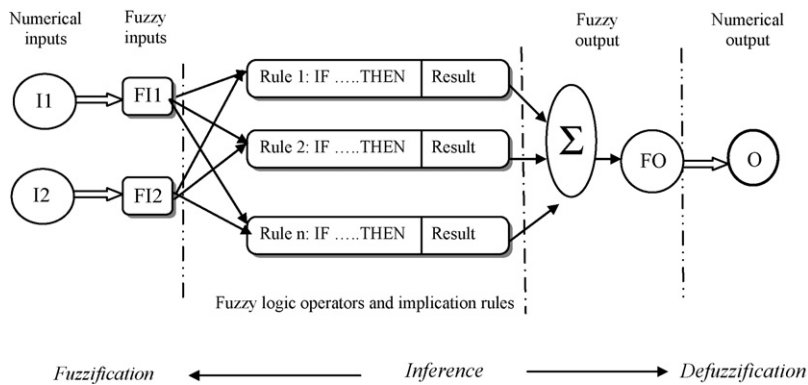
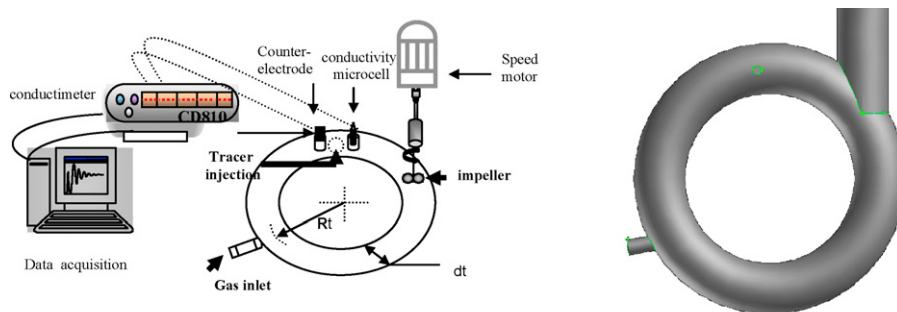


Fig. 2. Diagram of the MISO fuzzy system where: I1, I2 are respectively the  $N$  and  $U_g$ , FI and FI2 are the input fuzzy variables, FO is the output fuzzy variable, O is the reduced concentration output.

in Fig. 3. The flow is induced by a marine impeller mounted on the shaft. The impeller has three pitched blades (blade angle  $\alpha = 40^\circ$ ) with an outer diameter of 20 mm and an axis diameter of 45 mm, it is driven by a speed motor. The impeller rotational speeds are varied between 200 and 800 rpm in a single-phase flow. In two-phase systems the same impeller rotation speeds are kept. Experiments are performed within the range of superficial gas velocities 0.011–

0.085 cm/s. The analysis of the local liquid mixing information is carried out through the measurement of the liquid phase electrolytic tracer concentration evolution as described in Ref. [4]. The electrode and the counter-electrode used in this study are connected to the conductimeter. It is important to note that the diameter of the platinum wire used as electrode is less than 0.2 mm and its insertion in the reactor do not significantly distort the flow



Inner diameter of the tube ( $d_i$ ) : 23.4 mm  
 Bending of reactor radius ( $R_t$ ) : 51 mm  
 Mean length of the milli reactor ( $L_t$ ) : 325 mm  
 Reactor volume ( $V_t$ ) : 0.14 L

Fig. 3. Milli torus reactor and the experimental set-up [4].

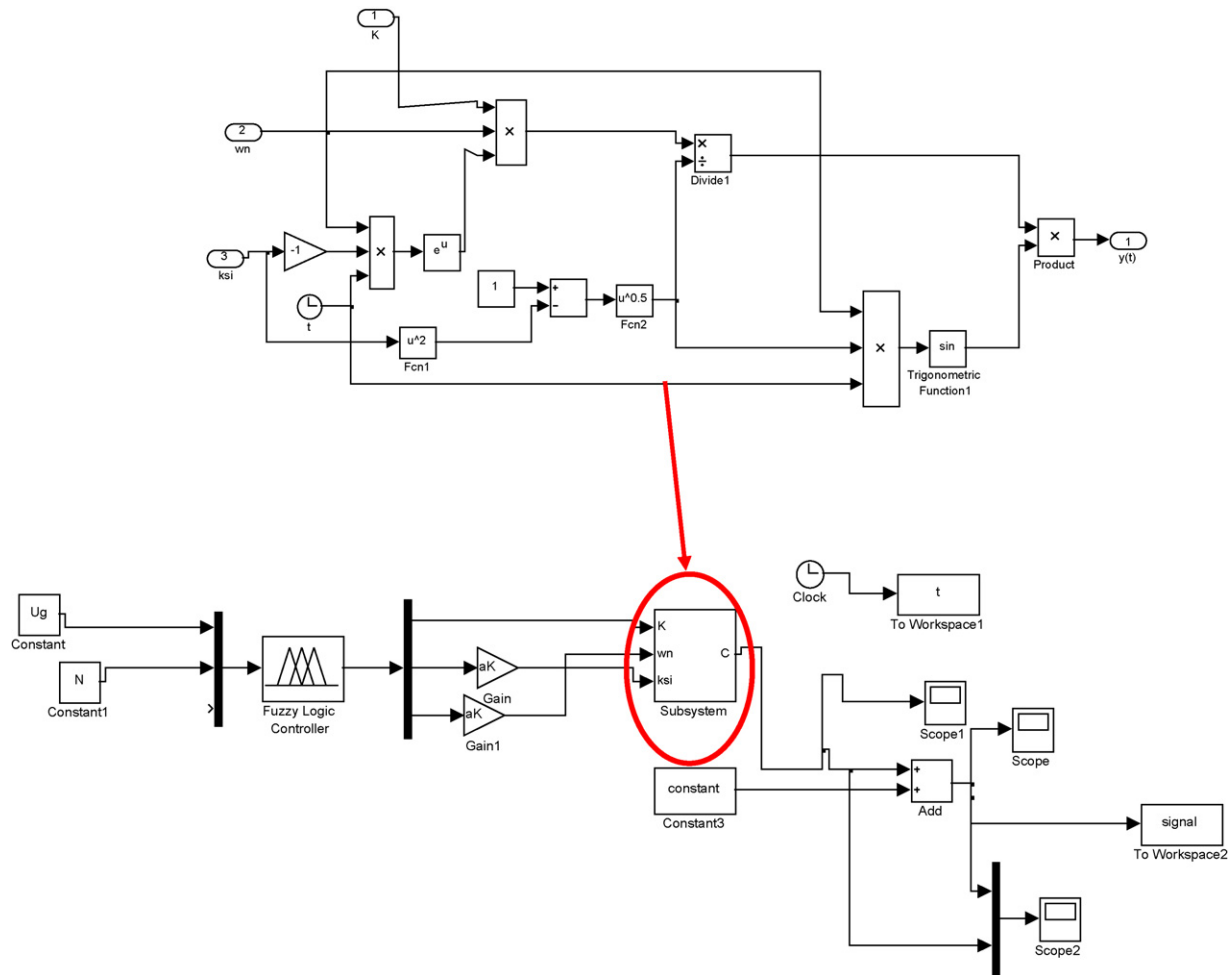


Fig. 4. Simulator scheme developed in Matlab environment for tracer response. Where  $k$ ,  $ksi$  and  $wn$  are the outputs of the fuzzy logic controller.

field. The acquisition of the amplified electrodes signal and the storage of data on a PC are realized via Labview.

## 4. Results and discussion

### 4.1. Simulation environment

Before building up the fuzzy system in Matlab7®/Simulink it is important to clarify the procedure:

- the first step consists in investigating the behaviour of tracer response
- the second step concerns the identification of the parameters to be varied: amplitude  $k$ , peak numbers  $wn$  and the damping  $ksi$ .

A simulator is developed under Matlab7/Simulink software package program and fuzzy logic Toolbox environment for modelling dynamic systems. The use of Matlab package makes the use of advanced design technique more feasible and simplifies the design process. It includes a series of block diagrams of inputs, fuzzy logic controller, subsystem and output (Fig. 4). To control the process, two inputs ( $N$ ,  $Ug$ ), which truly influence the output are entered. The controller is an entity which will enforce the desired behaviour through the manipulated variables ( $N$ ,  $Ug$ ) and corresponding rules. It appears appropriate because the behaviour of the system can be described using only linguistic variables. The parameters of this controller are tested alone, *i.e.* without subsystem before starting

the simulation, if our goal is not met, new predefined rules are necessary, if the goal is met, the controller is kept. Among its advantages is the simplification of the controller design. For this kind of simulator, a key point is to establish a mathematical model for the process to be controlled. Evolution of dynamic process depends not only on the values of inputs, but also upon the cumulated effects and the geometry of the system. For such a case the subsystem is based on the resolution of a second-order differential equation (Eq. (2)). The inputs and the output are related by a controller and a linear differential equation. For accuracy of results, we introduce Gain1, Gain 2 corresponding to  $wn$  and  $ksi$  respectively. The tracer response behaviour is automatically passed to the Matlab workspace and scope via (time, output signal).

### 4.2. Membership functions

A membership function is a curve that defines the distribution of truth variables around values which vary in the interval  $[0,1]$  by agreement.

There are several membership function types, the simplest in design based on little information, speed; efficiency and use are triangular and trapezoidal fuzzy membership functions [27]. So they are predominant in current applications of fuzzy set theory.

Generally, the membership functions and rules are determined from the RTD experiments of the process, the step response, etc. There are several triangular and trapezoidal membership functions used for the variable inputs ( $N$ ,  $Ug$ ), and outputs ( $k$ ,  $ksi$ ,  $wn$ ). Thus

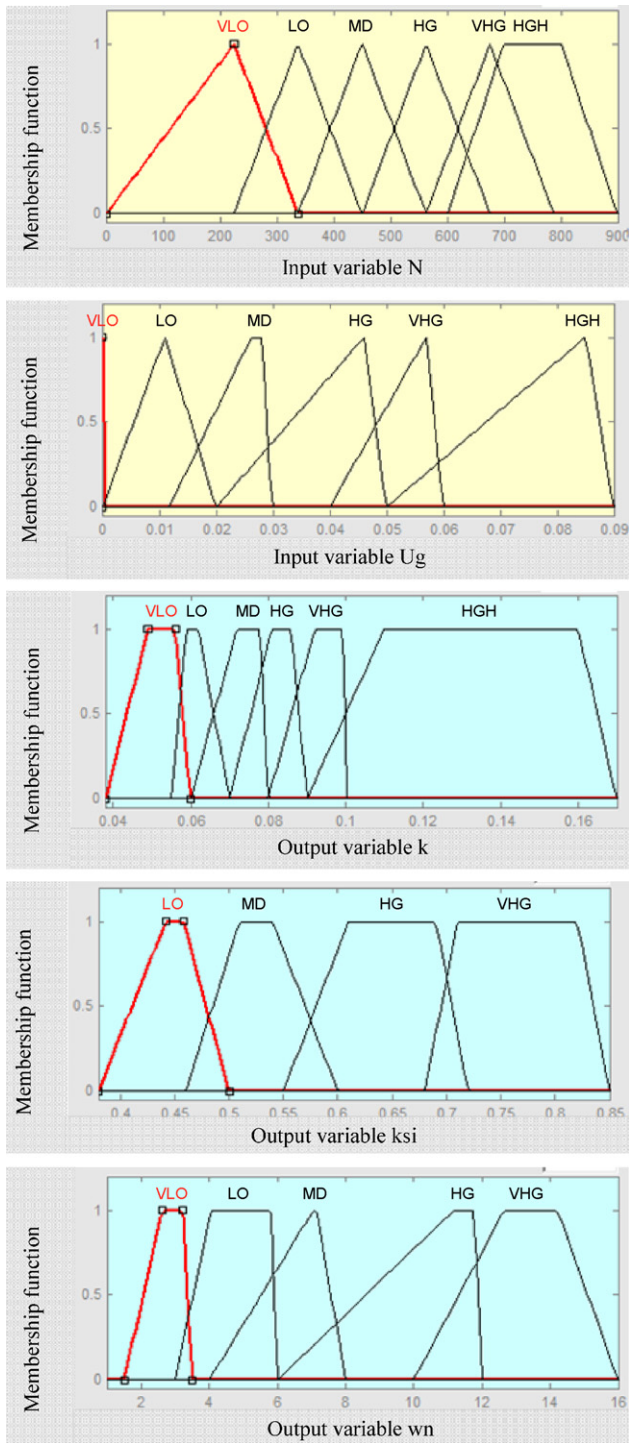


Fig. 5. Membership functions of the input and output fuzzy variables.

forty rules are defined in the universe of discourses of linguistic variables (Fig. 5). The more the number of membership functions is important the more classes are defined in the universe of discourses, leading to the increase of the sensitivity of the system for the variables mentioned above. For  $U_g$ , the membership function “zero” indicates a very low range of the superficial gas velocity, *i.e.* non-aerated conditions. Genovesi et al. [28] recommended the use of a symmetric form such as equilateral or isosceles triangle because it is easy to implement. In this paper, one trapezoidal membership function for each input is used, isosceles and irregular triangles are used also for input parameters, where their maxima correspond to

the actual inputs. For the outputs, the majority of the membership functions are chosen as trapezoidal functions instead of triangular functions because these functions are more appropriate to obtain the desired real response from the system. For example, when two very low (VLO) inputs ( $N$ ,  $U_g$ ) are entered, the  $k$  and  $wn$  variables are considered “fully” very low and  $ksi$  “fully” low. Some trapezoidal functions are wider compared to the others which mean that almost all the values belonging to the same intervals and sharing the same influence or the same behaviour. Note that the level of activation of each rule is computed with the minimum operator and the aggregation with the maximum operator.

### 4.3. Control surface

The shape of each membership function is determined using the fuzzy control surface or surface decision (Fig. 6) to produce the

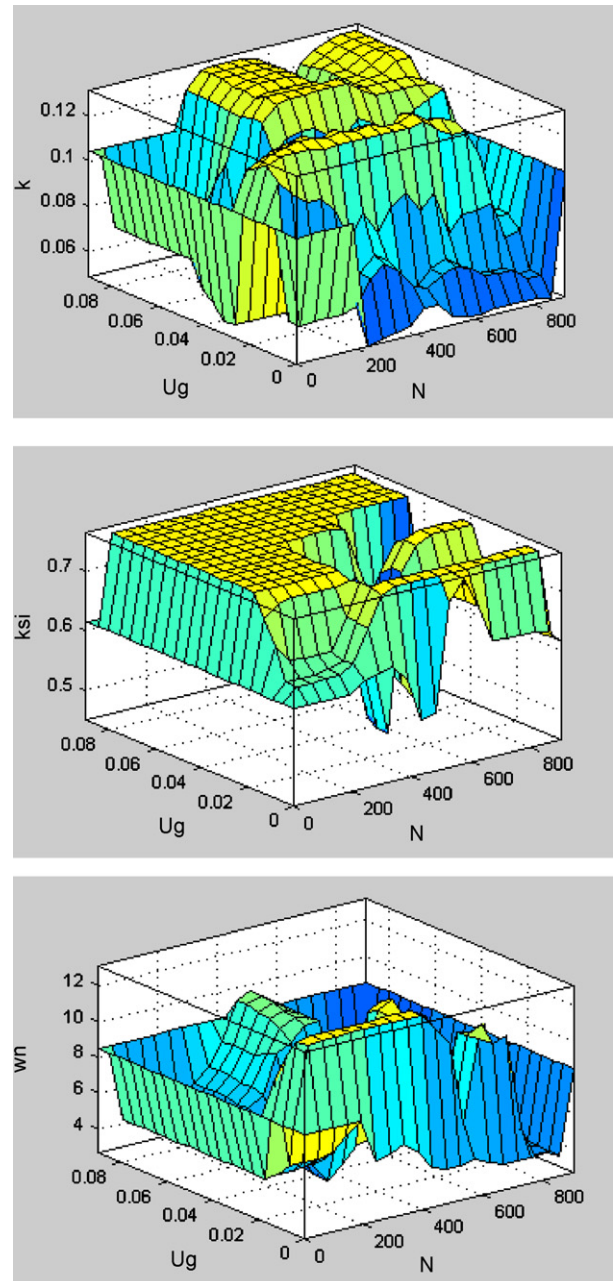


Fig. 6. Fuzzy control surface.

desired tracer response in the case of single and two-phase flow.  $N$  and  $U_g$  inputs to a controller are on the horizontal axes whereas  $(k, ksi$  and  $w_n)$  outputs of a controller are on the vertical axis. These figures show that the numerical output values depend on the inputs  $(N, U_g)$  and their combinations to the predefined rules are set by using logical operations for example AND. The first curve and the later one present hyperbolic paraboloid shelter surface even though the second gives fractal surface. For example, when  $U_g$  and  $N$  are at a maximum, the  $k$  is at a maximum.

#### 4.4. Tracer concentration variations

Simulation of the scalar mixing of a passive tracer is compared to the experimental data using a direct fit (Figs. 7 and 8). The classical model [29] is also plotted in the same figure for comparison. This model is proposed for modelling the mixing process in a turbine-stirred baffled tank, they divided the vessel into two regions with different degrees of mixing: the first one consisting of the impeller region and the second one is the remaining region of the vessel. In the circulation loop, mixing is not intense and is characterised by a dispersion coefficient and an average circulation liquid velocity. The Voncken's equation [29] is expressed by:

$$\frac{C(\theta)}{C_o} = \sqrt{\frac{Bo}{4\pi\theta}} \sum_{j=1}^{\infty} \exp\left[\frac{-Bo(j-\theta)^2}{4\theta}\right] \quad (8)$$

where  $Bo = V_L L_t / E_z$  is the Bodenstein number,  $C(\theta)$  is the tracer concentration inside the reactor,  $C_o$  is the final tracer concentration,  $\theta = t/t_c$  is the dimensionless time,  $j = Z/L_t$  is the dimensionless distance,  $L_t$  is the mean length of reactor,  $Z$  is the geometrical distance between injection and detection.

The relative height of the peaks and their number depend on the axial dispersion coefficient. For accurate simulation of mixing in the reactor, the equivalent output  $C/C_o$  is obtained by adding proportional gain ( $aK$ ). Gain values are obtained according to the system tracer response curve. In order to control the system much better it is preferable in some advanced controllers, to vary the gain parameters throughout the operation as reported by Çam [30]. For each simulation, a small time step 0.01 s is taken as data acquisition interval. In fact, it does not affect tracer dispersion. After that, the controller is adjusted to find its best value that permitted the best fit. It seems that the experimental tracer response curves fit quite well with those calculated from the models. Fuzzy logic model provides a good estimation of cycling time, which represents the extent of convective transport [24]. Nevertheless, the differences are only important during the first peak corresponding to about one-quarter of the entrainment of the tracer by the impeller or by both impeller rotation and incoming gas. The difference between the experimental dispersion and the simulated ones may be due to the underestimation of the dispersion of the tracer by the movement of the bubbles and the impeller just after injecting the tracer. In such case, the tracer is splitted in the axial direction by gas phase flow which is in ascendant mode while the impeller generating an axial downwards flow, i.e. the tracer is not predominantly dispersed by eddy diffusion. When the tracer reaches the impeller, it is dragged along the reactor until attaining the fully mixed conditions. Also the fuzzy logic model deals only with the physical description of the tracer response curves and does not take into account the existence of delay between injection and detection. The delay is defined by an implicit equation [1]. Then, the classical model and the fuzzy logic model give results which are comparable. When adding  $U_g$  variable to our system, the simulator provides more peaks to those observed in single-phase system depending to the flow regime observed. The fuzzy logic modelling gives a good dispersion and robustness against variations of system parameters.

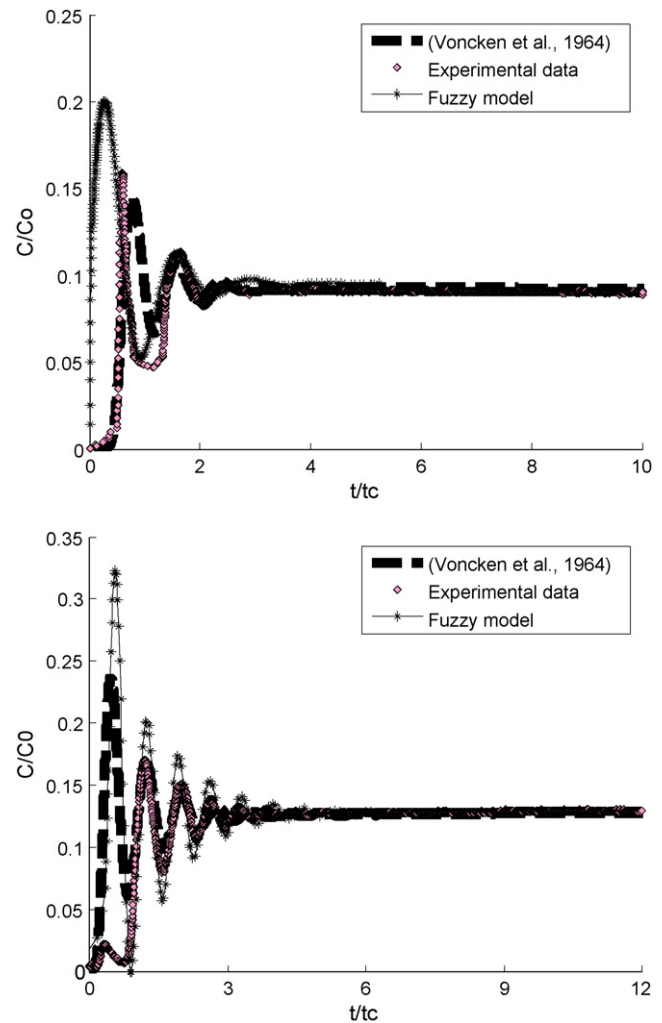


Fig. 7. Typical experimental and simulated exit-tracer concentration curves using fuzzy logic for  $N = 500$  rpm for (a) single-phase (RMSE = 0.18) (b) two-phase flow ( $U_g = 0.011$  cm/s) (RMSE = 0.13).

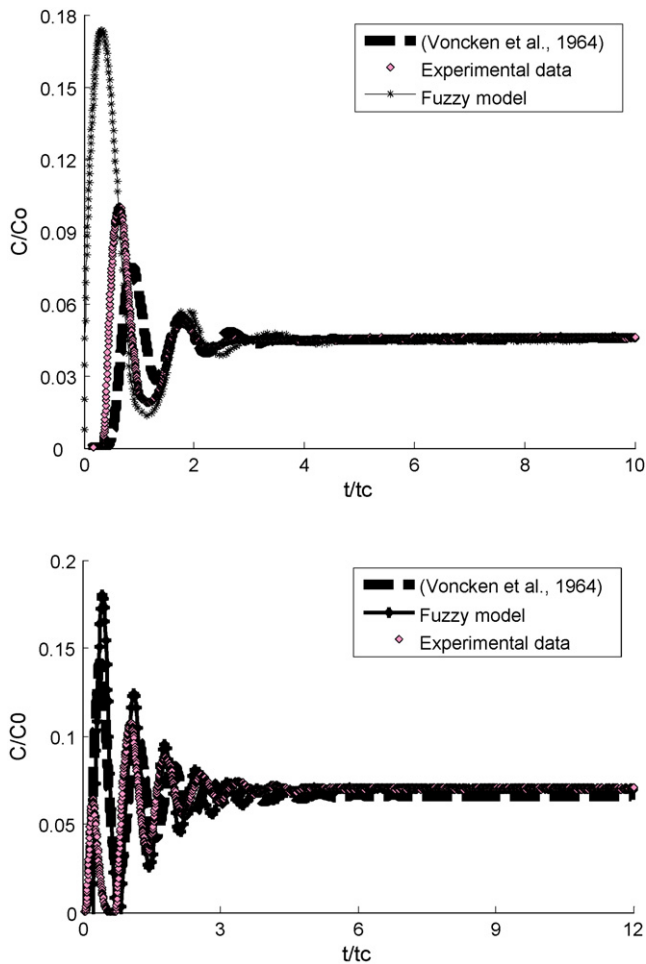
When modelling tracer response, an acceptable RMSE is obtained in the case of the fuzzy model. The mean squared error is calculated as follows:

$$RMSE = \left[ \frac{\sum_{i=1}^n (C_{imeas}(t) - C_{ical}(t))^2}{\sum_{i=1}^n (C_{imeas}(t))^2} \right]^{1/2} \quad (9)$$

where RMSE is the comparison of the experimental data with the one calculated by fuzzy logic model.

#### 4.5. Validity of the fuzzy model

In order to test the robustness of the proposed fuzzy logic controller and check the validity of the method against variations of system parameters, simulations are carried out for an extended range, i.e. outside the experimental range of variable  $N$  used for the establishment of the controller. It is tested for the impeller rotation speed  $N = 1000$  rpm and the superficial gas velocity  $U_g = 0.085$  cm/s. The latter remains the upper velocity studied in this paper because beyond this velocity, the collected signal is too noisy due to the bubbles hitting the microprobes despite the fact that the platinum microprobe is encased in glass tubing [4]. Every change in the input parameters has an influence on the output of  $C/C_o$ . In such case, corresponding rule is activated. Our objective is to test the developed simulator and to show its limits. Fig. 9 shows that the fuzzy logic

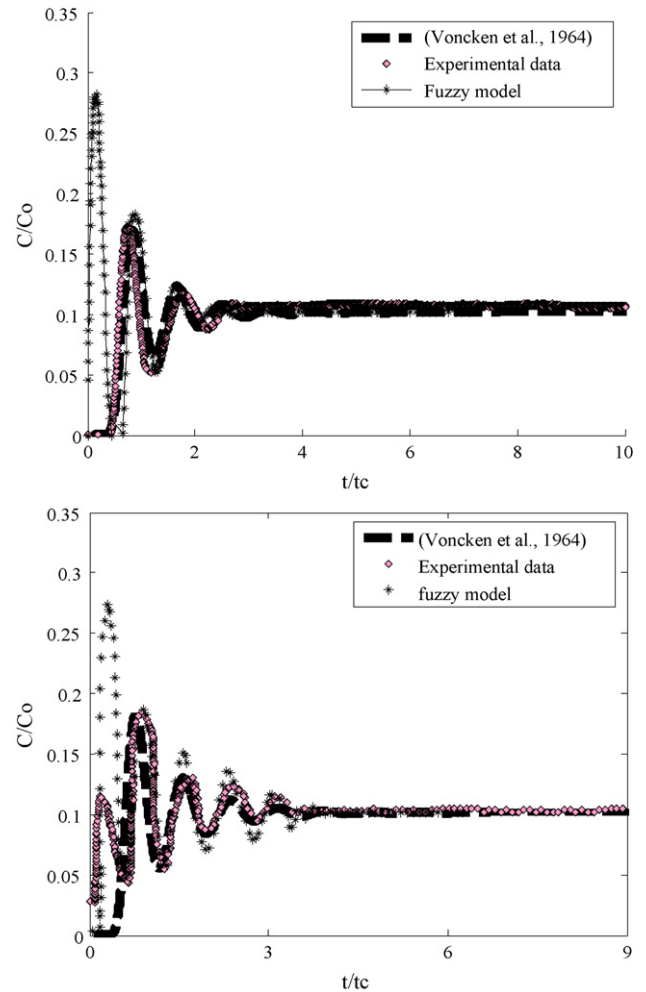


**Fig. 8.** Typical experimental and simulated exit-tracer concentration curves using fuzzy logic for  $N=800$  rpm for (a) single-phase (RMSE=0.20)(b) two-phase flow ( $U_g=0.028$  cm/s) (RMSE=0.15).

model fits quite well the experimental data and does not change while the system parameters are extended. These results confirm the robustness of the proposed controller except for the first peak. It can be considered as a suitable technique for non-linear and time-variation systems. These curves clearly demonstrate that the fuzzy logic modelling is robust and gives similar results as the classical model. It can detect more peaks quickly when we just enter set of inputs, whereas the classical model is estimated as a fitting parameter to obtain the best fit between the experimental data and the Voncken's equation [29]. In the fuzzy logic model, the controller is characterised by three physical parameters  $k$ ,  $\omega_n$  and  $\text{ksi}$  whereas in classical model the liquid dispersion is described only by two parameters the average circulation liquid velocity and the Bodenstein number  $Bo$ . The latter leads to the describing of the hydrodynamic behaviour of the fluid flow in the reactors. Patwardhan and Joshi [31] reported that a single parameter  $Bo$  cannot capture the mixing process occurring in the tank.

#### 4.6. Mixing time

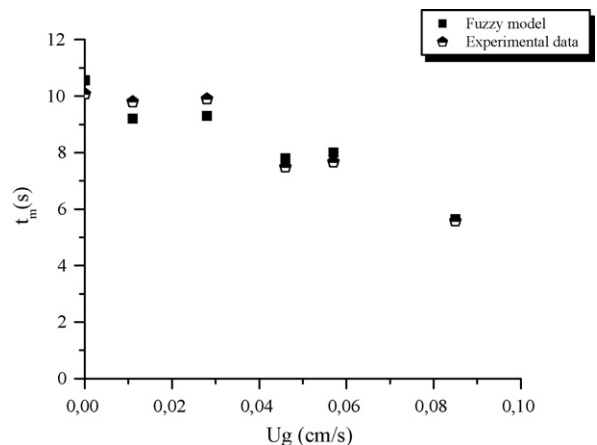
In order to test the accuracy of the fuzzy logic model, the evolution and the comparison of the values of measured mixing times and values calculated by the model are reported in Figs. 10 and 11. Mixing time has the disadvantage that it is specific to the reactor design and scale, but it is easy to measure and understand. It is evaluated after several tracer passages as the sum of the mixing in



**Fig. 9.** Exit-tracer concentration curves using fuzzy logic for  $N=1000$  rpm for (a) single-phase (RMSE=0.22) (b) two-phase flow ( $U_g=0.085$  cm/s) (RMSE=0.15).

the subsequent passages, *i.e.* the deviation of the envelope of the maxima [25].

The predicted mixing times agree well with the experimental data under the given conditions. It decreases with the increase of the superficial gas velocity which promotes better mixing process than in non-aerated conditions. Indeed, the predicted mixing time is about 5.65 s for  $N=10\text{ s}^{-1}$  and for the superficial gas velocity



**Fig. 10.** Evolution of the mixing time with superficial gas velocity and for  $N=10\text{ s}^{-1}$ .



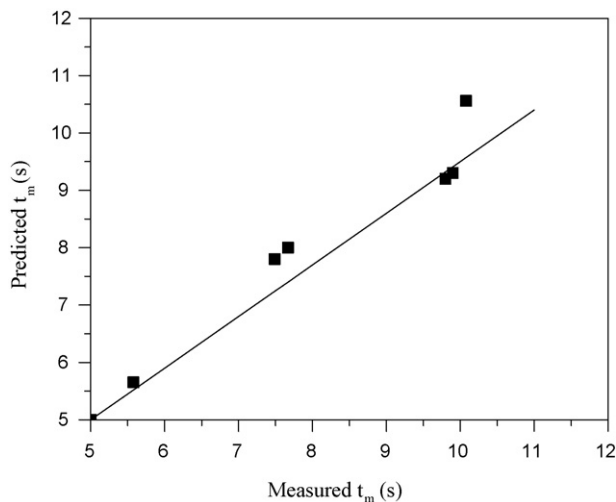


Fig. 11. Comparison between the predicted and the measured  $t_m$  values.

$U_g = 0.085$  cm/s, while the measured one is equal to about 5.58 s under the same conditions. The error on the prediction of  $t_m$  is generally lower than 6%. All these results show that the present fuzzy logic model seems to adequately capture the main features of fluid dynamics and mixing of single and two-phase flow. Note that the fuzzy logic model is a concept not limited to only to a specific mixing system design. Nevertheless, the proposed simulator can be generalized to describe mixing processes in other configurations under aerated and non-aerated conditions.

## 5. Conclusion

The aim of this study is to develop a simulator, for modelling tracer response in the case of single and two-phase systems in milli loop geometry. The main advantage of this simulator is that it allows the use of multiple inputs, and its ability to describe the controller behaviour with several rules defined by a few linguistic variables. The rules may be considered as an effective mean to transmit knowledge between humans, to make and to justify decisions [32]. It takes into account the information describing the experimental characteristics of the tracer responses, such as peaks number, response time for the first peak, circulation time, the equilibrium concentration, etc. This model fits well the data with acceptable error.

The above-developed simulator can be used in the case of single-phase and two-phase systems (gas and liquid involved in the experiments) in different configurations such as torus geometry, airlift and stirred reactors in batch mode and for scale-up. It is based on a second-order differential equation. Also, it can be extended for multiphase systems. In such case the number of inputs depends on the process complexity, in certain case the subsystem requires modification. The robustness of the rules is proved by mean of simulation experiments. Fuzzy logic can handle one and two-phase flow but more testing against data at large scale is required to build confidence.

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